

METHOD FOR DETERMINING AREA OF PARAMETRIC RESONANCES OF WHEELSET OF RAILWAY VEHICLE

Viktor Nikolaev¹, Janat Musayev², Algazy Zhauyt³, Aizhan Makhanova², Zhanar Mukatanova³, Anar Kabytkarim³, Elnur Kozhakmetov³

¹Omsk State Transport University, Russia; ²Academy of Logistics and Transport, Kazakhstan;

³Almaty University of Power Engineering and Telecommunication, Kazakhstan

nikolaevVA@omgups.ru, ali84jauit@mail.ru, maxanova.2012@mail.ru, zh.mukatanova@aes.kz, a.kabytkarim@aes.kz, e.kozhakmetov@aes.kz

Abstract. Resonance phenomena take place not only in the field of sound and electricity, but also in mechanics, optics, physics, it is known that resonance can be, for example, a cause of destruction of a bridge, tunnel under the action of periodic forces, train derailment due to intense vibrations of rolling stock. It is shown that in mechanical systems with one degree of freedom there can be only simple parametric resonance and the number is countable. The article proposes a method for determining areas of parametric resonances of a railway carriage wheel pair. On the basis of the trigonometric method main areas of parametric resonance of railway carriage wheel pair have been established. To study the vertical displacements of units and parts of the vehicle, the authors proposed a design scheme for a conditional uniaxial vehicle. The authors found that for a given range of parametric perturbation coefficient variation it is enough to consider only one harmonic, and secondly, the width of the resonance zone increases with growth of the parametric perturbation coefficient. The performed studies have established that the longitudinal uneven elasticity of the railway track creates sufficiently large transfer accelerations acting on the rolling stock units, and the maximum amplitude of the wheelset bouncing due to the longitudinal uneven elasticity of the track is in order of magnitude higher than the same indicator from the geometric unevenness on the rail tread surface. The results obtained in establishing the areas of parametric resonances of the wheelset of a railway vehicle will be useful in developing recommendations for normalizing the values of the uneven elasticity of the railway track.

Keywords: wheelset, railway vehicle, differential equations, general physics, vibration.

Introduction

An analysis of scientific and technical literature shows that a dynamic system of oscillatory type has some natural frequencies. The coincidence with the natural frequencies of external perturbation, acting on a mechanical, electrical or electromechanical object, leads to the fact that resonance mode is established in the system. Consequently, this causes increased loads on elements of the system [1]. The stated idea is correct only under one condition, the mathematical model such as a dynamic system is represented by a system of ordinary differential equations with constant coefficients [2]. It is most well established in the dynamics of rolling stock of railways. Indeed, the very concept of resonance is the most important and basic, a well-known concept from the school course of general physics. Everyone also knows very well that resonance phenomena occur not only in the field of sound and electricity, but also in mechanics, optics, and physics. So, resonance can be, for example, the cause of destruction of a bridge, a tunnel under the action of periodic forces, a train derailment due to intense oscillations of bouncing, galloping, drift, side roll, wobble or drift, breakage of shafts at critical rotational speeds, etc. The study of the occurrence of resonance in the interaction of traditional and high-speed railway vehicles and the track is found in the works of various authors [1-2], which are related to the study of the causes of appearance of self-excited oscillations, parametric resonance caused by an increase in the speed of movement, imbalance of the wheelset and other operational and technological factors.

Materials and methods

The evolution concept of resonance is described in detail by N. D. Papaleksi [3]. The role of resonance in science is extremely great, and in real life we almost constantly encounter one or another manifestations of resonance. It is very important to have a deep and clear understanding of what we mean by resonance. It is known that if mechanical system some parameter changes over time, then in it, under certain conditions, a parametric resonance can occur [5]. The equations of parametric vibrations of linear systems with finite number degrees of freedom in general case are represented as:

The calculated scheme of oscillations of the unsprung mass of the vehicle is presented in the form of a diagram in Fig. 1.

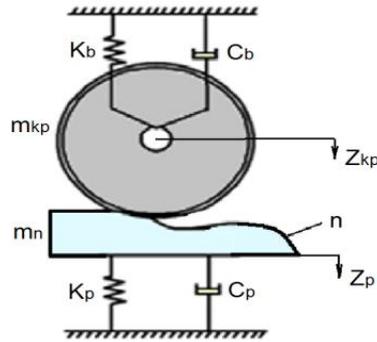


Fig. 1. Calculation scheme of oscillations of the unsprung mass of the crew

To find the equation for the oscillation of the unsprung mass of the vehicle (see Fig. 1), we use the Lagrange equations of the second kind. Therefore, we calculate the kinetic, potential energy and dissipative function.

The kinetic energy of the wheelset is:

$$T = \frac{1}{2} (m_{kp} + m_p) \dot{z}_{kp}^2. \quad (1)$$

The potential energy of the crew is defined as follows:

$$\Pi(x) = \frac{1}{2} [K_b + K_p(x)] z_{kp}^2 - (P_{kp} + m_{kp}g) z_{kp}. \quad (2)$$

It, of course, will be a function of the position of the wheelset of the vehicle in the rail track since the rigidity of the track depends on this position [6].

The dissipative function can be found only if the friction forces in the axle box suspension and the track are in the nature of viscous friction. Therefore, we believe that the friction in the axle box suspension stage of the rolling stock was linearized in advance. Then, according to the method of calculating the equivalent coefficient of viscous friction, we have:

$$\Phi = \frac{1}{2} (C_b + C_p) \dot{z}_{kp}^2, \quad (3)$$

- where m_{kp} – mass of the wheelset of the vehicle;
 m_p – “reduced” mass of the track;
 K_b – stiffness of the axle box suspension of the vehicle;
 K_p – rigidity of the railway track in the vertical plane, calculated by Eq. (2),
 P_{kp} – weight of the vehicle per one wheel set,
 C_b – coefficient of viscous friction in axle box suspension,
 C_p – coefficient of viscous friction on the way, the absolute speed of bouncing of the wheelset of the vehicle;
 z_{kp} – absolute displacement of the wheelset in the vertical plane;
 z_p – bouncing of the reduced mass of the railway track.

We need taking the derivatives from the functions written above, and substituting the obtained values into the Lagrange equation of the second kind, which in our case has the form:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{z}_{kp}} \right) - \frac{\partial T}{\partial z_{kp}} + \frac{\partial \Phi}{\partial \dot{z}_{kp}} + \frac{\partial \Pi}{\partial z_{kp}} = 0. \quad (4)$$

We find the differential equation for fluctuations in the unsprung mass of the vehicle:

$$m \ddot{z}_{kp} + b \dot{z}_{kp} + K(x) z_{kp} - P_{rail} = 0, \quad (5)$$

- where $m = m_{kp} + m_p$ – reduced mass of the wheel set;
 $C = C_b + C_p$ – reduced coefficient of viscous friction in the system “crew – track”;
 $K(x) = K_b + K_p(x)$ – reduced stiffness considered mechanical system;

P_{rail} – pressure of the wheelset on the rails.

This differential equation describes if it is permissible to put it this way the free vibrations of the bouncing of the wheelset as it rolls along a path that is unequal in length in the absence of geometric irregularities on the rail surfaces [7]. We note here that if the path were unequal elastic in length, and the initial conditions were trivial, then the wheelset would simply move along the path without oscillation. But this is an absolutely ideal case, since in reality this does not happen, because there will always be some imperfection in the path design.

To simplify Eq. (6) (where $P_{rail} = const$), we replace z_{kp} by $q + f$, here q is the bouncing of the wheelset relative to the equilibrium position, which is established from the equality:

$$K(x)f(x) - P_{rail} = 0, \quad (6)$$

$$f(x) = \frac{P_{rail}}{K(x)}.$$

If the railway track were equally elastic in length, then the value $f(x)$ would not depend on the position of the vehicle in the track but would be constant. But in our case, we will assume that the vehicle moves along the railway track at a constant speed, then we have that $x = Vt$, here V is the velocity of the vehicle in $m \cdot s^{-1}$. Therefore, we get:

$$f(x) = \frac{P_{rail}}{K_b + K_p \left(1 - 2\mu \sin \frac{2\pi \mathcal{G}t}{l_s} \right)} = \frac{P_{rail}}{K_b + K_p - 2\mu K_p \sin 2\Omega t} = \frac{P_{rail}}{K_b + K_p} \cdot \frac{1}{1 - 2\mu \frac{K_p}{K_b + K_p} \sin 2\Omega t}. \quad (7)$$

Let us introduce a new notation: $f_0 = P_{rail}/(K_b + K_p)$ – average static deflection of the railway track under constant pressure of the wheel set; $\Omega = \pi \mathcal{G}/l_s$ – frequency of parametric excitation; l_s – distance between the middle of the sleepers (usually determined by the diagram of the sleepers, in our case $l_s = 54$ cm); $\varepsilon = \mu K_p/(K_b + K_p)$ – small parameter (dimensionless value) and rewrite Eq. (8) as follows:

$$f(t) = \frac{f_0}{1 - 2\varepsilon \sin 2\Omega t}, \quad (8)$$

Further, after simple transformations, we will find the following differential equation for bouncing of the wheelset of the vehicle when moving along a railway track that is not uniformly elastic in a vertical plane:

$$m\ddot{q} + b\dot{q} + K(t)q = -m\ddot{f} - b\dot{f}, \quad (9)$$

where $K(t) = (K_b + K_p)(1 - 2\varepsilon \sin 2\Omega t)$ – harmonic change in the rigidity of the mechanical system “crew – track”.

The differential Eq. (9) is characterized by both a multiplicative perturbation (this is a change, in our case, harmonic, of the rigidity of a mechanical system, i.e. a path), and an additive effect, which is on the right side of Eq. (9), characterizing the translational motion of the system. We emphasize that the generalized coordinate q is measured by us from the position of the static equilibrium of the system at each point of the railway track, which is constantly changing. And since the differential equation has a right-hand side, both forced and parametric oscillations will inevitably develop in the mechanical system [7].

The design scheme of a conditional uniaxial vehicle is shown in Figure 2. Let us agree that the vehicle can only perform vertical movements, in other words, it can only bounce. Therefore, the z axis is directed vertically upwards, and is also indicated z_k bouncing of the sprung weight of the vehicle, z_{kp} – bouncing of its wheel set, z_p – bouncing of the “reduced” mass of the railway track, P_{rail} – pressure of the wheelset on the rails, K_b and C_b – stiffness and coefficient of viscous friction of the axle box suspension of the vehicle, K_p and C_p – stiffness and coefficient of viscous friction in the way.

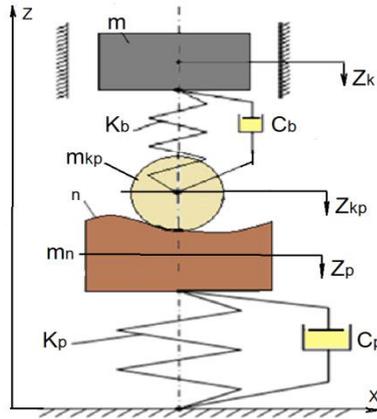


Fig 2. Calculation scheme of a conditional single-axle vehicle

Trigonometric method and results

Let us start determining the main region of parametric resonance of the wheelset of a railway vehicle. Assuming that solution boundary is approximately equal (we take into account only the first and third harmonics):

$$q \approx A_1 \sin \Omega t + B_1 \cos \Omega t + A_3 \sin 3\Omega t + B_3 \cos 3\Omega t . \tag{10}$$

Let us take the second derivative expression and substitute the whole into the following differential equation, describing the behavior of the wheelset in the conservative case:

$$\ddot{q} + k_0^2(1 - 2\varepsilon \sin 2\Omega t)q = 0 . \tag{11}$$

After simple transformations, taking into account trigonometric equations and discarding the fifth harmonic, we obtain:

$$\begin{aligned} & (k_0^2 - \Omega^2)A_1 \sin \Omega t + (k_0^2 - \Omega^2)B_1 \cos \Omega t + (k_0^2 - 9\Omega^2)A_3 \sin 3\Omega t + (k_0^2 - 9\Omega^2)B_3 \cos 3\Omega t - \\ & - \varepsilon k_0^2 A_1 (\cos \Omega t - \cos 3\Omega t) - \varepsilon k_0^2 B_1 (\sin \Omega t - \sin 3\Omega t) - \varepsilon k_0^2 A_3 \cos \Omega t + \varepsilon k_0^2 B_3 \sin \Omega t \approx 0 . \end{aligned} \tag{12}$$

From here we can find the determinant corresponding to the nontrivial solution system Eq. (12), while only unknown in frequency of the parametric perturbation:

$$\begin{vmatrix} k_0^2 - \Omega^2 & -\varepsilon k_0^2 & 0 & \varepsilon k_0^2 \\ -\varepsilon k_0^2 & k_0^2 - \Omega^2 & -\varepsilon k_0^2 & 0 \\ 0 & -\varepsilon k_0^2 & k_0^2 - 9\Omega^2 & 0 \\ \varepsilon k_0^2 & 0 & 0 & k_0^2 - 9\Omega^2 \end{vmatrix} = 0 . \tag{13}$$

This can be an easily reduced problem of calculating eigenvalues of a certain matrix. To do this, we divide the third and fourth lines by 9, as a result we have:

$$\begin{vmatrix} k_0^2 - \Omega^2 & -\varepsilon k_0^2 & 0 & \varepsilon k_0^2 \\ -\varepsilon k_0^2 & k_0^2 - \Omega^2 & -\varepsilon k_0^2 & 0 \\ 0 & -\frac{1}{9}\varepsilon k_0^2 & \frac{1}{9}k_0^2 - \Omega^2 & 0 \\ \frac{1}{9}\varepsilon k_0^2 & 0 & 0 & \frac{1}{9}k_0^2 - \Omega^2 \end{vmatrix} = 0 . \tag{14}$$

Next, we will determine by numerical methods using a computer the boundaries of the zone of demultiplication (principal) resonance. First, we obtain approximate formulas for their calculation. This will give us the opportunity to evaluate their accuracy. So, let us omit the third harmonic in the solution of Eq. (10), which is equivalent to crossing out the third and fourth rows and columns, then we have:

$$\begin{vmatrix} k_0^2 - \Omega^2 & -\varepsilon k_0^2 \\ -\varepsilon k_0^2 & k_0^2 - \Omega^2 \end{vmatrix} = 0 . \tag{15}$$

It is not difficult to establish that:

$$\frac{\Omega}{k_0} = \sqrt{1 \pm \varepsilon} . \tag{16}$$

Approximate boundaries of the first (main) region parametric resonance, calculated from Eq. (16) are shown in Figure 3. Taking into account the third harmonic, i.e. search for eigenvalues of the matrix Eq. (14), gives the result shown in Figure 4. Further, taking into account in solution Eq. (10) the fifth harmonic, we represent the coordinate q in the form of the following set of harmonics:

$$q \approx A_1 \sin \Omega t + B_1 \cos \Omega t + A_3 \sin 3\Omega t + B_3 \cos 3\Omega t + A_5 \sin 5\Omega t + B_5 \cos 5\Omega t . \tag{17}$$

Expression of substitution Eq. (17) in Eq. (11) gives:

$$\begin{aligned} & (k_0^2 - \Omega^2)A_1 \sin \Omega t + (k_0^2 - \Omega^2)B_1 \cos \Omega t + (k_0^2 - 9\Omega^2)A_3 \sin 3\Omega t + (k_0^2 - 9\Omega^2)B_3 \cos 3\Omega t + \\ & + (k_0^2 - 25\Omega^2)A_5 \sin 5\Omega t + (k_0^2 - 25\Omega^2)B_5 \cos 5\Omega t - \varepsilon k_0^2 A_1 (\cos \Omega t - \cos 3\Omega t) - \\ & - \varepsilon k_0^2 B_1 (\sin \Omega t - \sin 3\Omega t) - \varepsilon k_0^2 A_3 \cos \Omega t + \varepsilon k_0^2 B_3 \sin \Omega t + \varepsilon k_0^2 A_5 \cos 5\Omega t - \varepsilon k_0^2 B_5 \sin 5\Omega t - \\ & - \varepsilon k_0^2 A_5 \cos 3\Omega t - \varepsilon k_0^2 B_5 \sin 3\Omega t \approx 0 . \end{aligned} \tag{18}$$

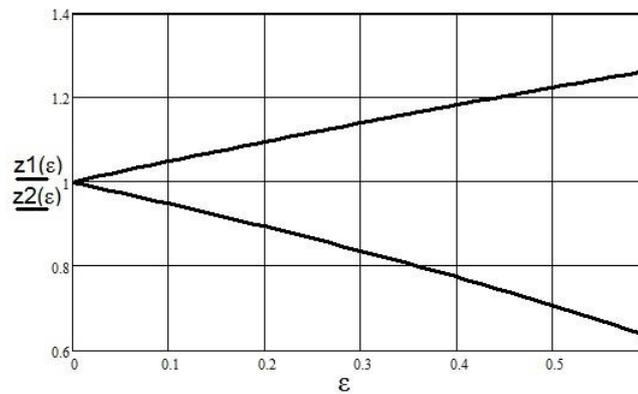


Fig. 3. Main area of parametric resonance of the vehicle wheelset

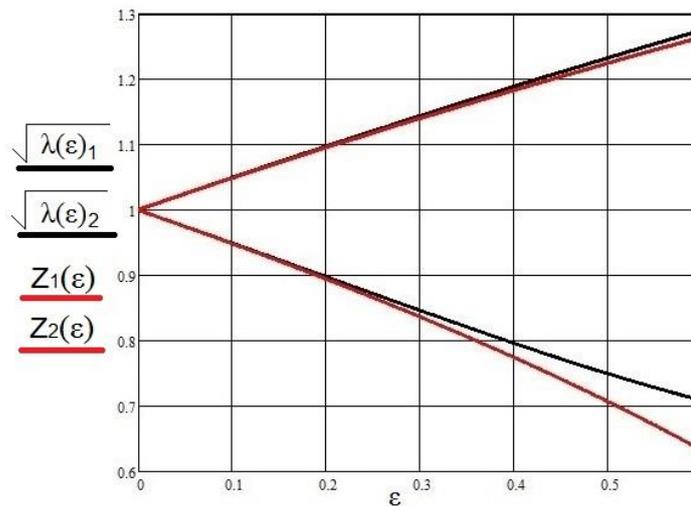


Fig. 4. Refined (by taking into account the third harmonic) area of the main parametric resonance of the wheel pair of the vehicle: the red dotted lines are the approximate formula (7), the black lines are when the third harmonic is taken into account

This relation must be satisfied at any time, so it is necessary to equate all the coefficients at $\sin \Omega t$, $\cos \Omega t$, $\sin 3\Omega t$, $\cos 3\Omega t$, $\sin 5\Omega t$ and $\cos 5\Omega t$. We are interested in a nontrivial solution of the resulting

system of algebraic equations. This leads to determinant in which the only unknown frequency is the frequency of the parametric perturbation:

$$\begin{vmatrix} k_0^2 - \Omega^2 & -\varepsilon k_0^2 & 0 & \varepsilon k_0^2 & 0 & 0 \\ -\varepsilon k_0^2 & k_0^2 - \Omega^2 & -\varepsilon k_0^2 & 0 & 0 & 0 \\ 0 & -\varepsilon k_0^2 & k_0^2 - 9\Omega^2 & \varepsilon k_0^2 & 0 & \varepsilon k_0^2 \\ \varepsilon k_0^2 & 0 & \varepsilon k_0^2 & k_0^2 - 9\Omega^2 & -\varepsilon k_0^2 & 0 \\ 0 & 0 & 0 & -\varepsilon k_0^2 & k_0^2 - 25\Omega^2 & 0 \\ 0 & 0 & \varepsilon k_0^2 & 0 & 0 & k_0^2 - 25\Omega^2 \end{vmatrix} = 0. \quad (19)$$

By dividing the third and fourth lines by 9 and the fifth and sixth lines by 25, we arrive at the standard matrix eigenvalue problem. The results of the calculations are shown in Figure 5.

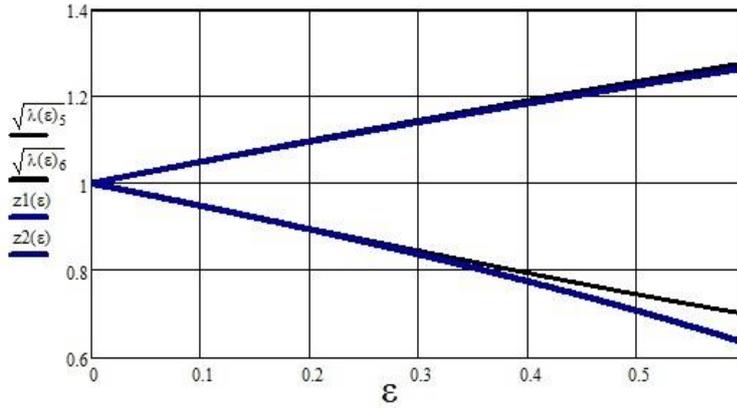


Fig. 5. Region of the main parametric resonance system, taking into account the third and fifth harmonics

The analysis of the results presented in Figures 3-5 allows us to draw preliminary conclusions: 1) for a given range of change in the parametric perturbation coefficient, it is quite sufficient to take into account only one harmonic; 2) the width resonant zone increases with an increase in the parametric perturbation coefficient [8].

To estimate the influence of dissipative forces on boundaries, we supplement our system with Eq. (11) viscous friction. And we will continue to look for the solution at the boundaries of the main parametric resonance in the trigonometric Eq. (10). Then the substitution Eq. (10-11) after the necessary transformations will give:

$$\begin{aligned} & (k_0^2 - \Omega^2)(A_1 \sin \Omega t + B_1 \cos \Omega t) + (k_0^2 - 9\Omega^2)(A_3 \sin 3\Omega t + B_3 \cos 3\Omega t) + \\ & + 2n\Omega(A_1 \cos \Omega t + B_1 \sin \Omega t) + 6n\Omega(A_3 \cos 3\Omega t - B_3 \sin 3\Omega t) - \varepsilon k_0^2(A_1 \cos \Omega t + B_1 \sin \Omega t) + \\ & + \varepsilon k_0^2(A_1 \cos 3\Omega t - B_1 \sin 3\Omega t) - \varepsilon k_0^2(A_3 \cos \Omega t - B_3 \sin \Omega t) \approx 0. \end{aligned} \quad (20)$$

Since this equation must be satisfied at any time, we equate to zero the coefficients at $\sin \Omega t$, $\cos \Omega t$, $\sin 3\Omega t$ and $\cos 3\Omega t$. Next, we formulate the condition for the absence of a trivial solution (otherwise parametric oscillations will not arise). As a result of the above transformations, we obtain:

$$\begin{vmatrix} k_0^2 - \Omega^2 & -(2n\Omega + \varepsilon k_0^2) & 0 & \varepsilon k_0^2 \\ (2n\Omega - \varepsilon k_0^2) & k_0^2 - \Omega^2 & -\varepsilon k_0^2 & 0 \\ 0 & -\varepsilon k_0^2 & k_0^2 - 9\Omega^2 & -6n\Omega \\ \varepsilon k_0^2 & 0 & 6n\Omega & k_0^2 - 9\Omega^2 \end{vmatrix} = 0. \quad (21)$$

In this secular equation, only one unknown is the parametric excitation frequency Ω of our railway vehicle [9]. In principle, after tedious transformations, one can find a polynomial in Ω and calculate its roots. For convenience, we discard the third harmonic and construct approximate formulas boundary of

the main region of parametric instability (or resonance). To do this, we cross out the third and fourth rows and columns and instead of Eq. (21) we have:

$$\begin{vmatrix} k_0^2 - \Omega^2 & -(2n\Omega + \varepsilon k_0^2) \\ (2n\Omega + \varepsilon k_0^2) & k_0^2 - \Omega^2 \end{vmatrix} = 0. \tag{22}$$

The result found is shown in Figure 6. Figure 6 shows that friction cuts off the main region of demultiplication resonance from the y-axis. However, this does not limit the resonant amplitudes, which tend to infinity over time [10]. This has been pointed out by the authors above. If we assume that the dimensionless coefficient of viscous friction δ in the axle box suspension vehicle is equal to 0.2, then the critical coefficient of parametric excitation will be approximately 0.392. Therefore, in order for the parametric resonance to develop in the vehicle, the parametric action coefficient must be greater than the reduced critical value [11]. Let the average rigidity railway track be equal to 5000 tf·m⁻¹, and the coefficient of parametric perturbation 0.4. Under such conditions, the difference between stiffness of the track above the sleeper and middle of the inter sleeper box should be equal to 4000 tf·m⁻¹. However, in the experiments to determine the modulus of elasticity of the track or its rigidity such a difference was not observed. This factor has a positive effect on the dynamics of the railway vehicle [12-13].

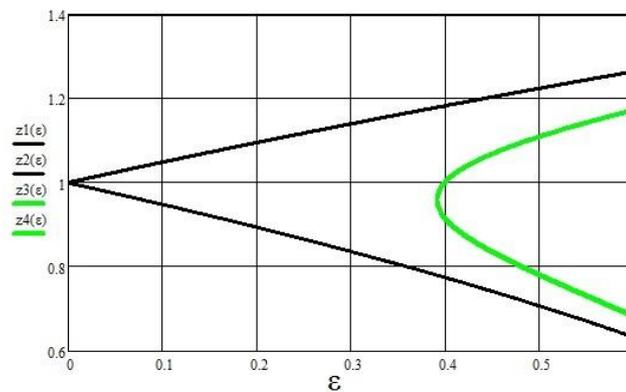


Fig. 6. Main area of parametric resonance wheel pair of the vehicle, taking into account viscous friction in its axle box suspension

Figures 7-9 below show the integral graphs of the displacement amplitudes of the bouncing of the wheel pair of the vehicle from the action of all disturbances for three movement speeds of 10, 20 and 30 m·s⁻¹, two multiplicative effect coefficients of 0.2 and 0.4 and two phase shifts between disturbances 45° and -45°.

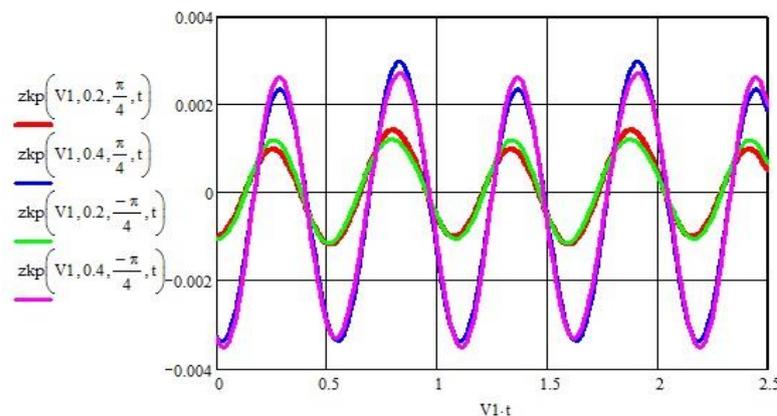


Fig. 7. Graph of bouncing of the wheelset of the vehicle at a speed of 10 m·s⁻¹ for two coefficients of parametric perturbation 0.2 and 0.4 and two phase shift angles between the effects of 45° and -45° (the abscissa is the movement of the vehicle along the path in m)

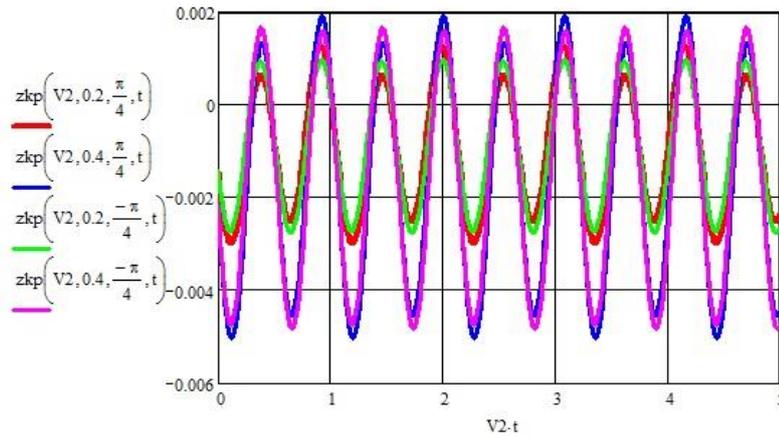


Fig. 8. Graph of bouncing of the wheelset of the vehicle at its speed of $20 \text{ m}\cdot\text{s}^{-1}$ for two coefficients of parametric perturbation 0.2 and 0.4 and two phase shift angles between the effects of 45° and -45° (the abscissa is the movement of the vehicle along the path in m)

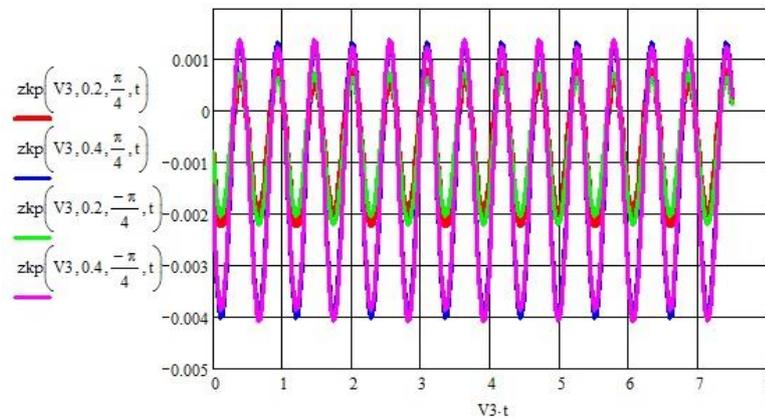


Fig. 9. Graph of bouncing of the wheelset of the vehicle at its speed of $30 \text{ m}\cdot\text{s}^{-1}$ for two coefficients of parametric perturbation 0.2 and 0.4 and two phase shift angles between the effects of 45° and -45° (the abscissa is the movement of the vehicle along the path in m)

Conclusions

From the above, the following can be concluded:

1. The parametric system in some sense resembles in its behavior a nonlinear system;
2. The angle of phase shift between disturbances plays a significant role due to the fact that it either increases or decreases the amplitude of the bouncing oscillations of the wheelset;
3. The main harmonic resonates at speeds from 10 to $27 \text{ m}\cdot\text{s}^{-1}$ (from $36 \text{ km}\cdot\text{h}^{-1}$ to $97.2 \text{ km}\cdot\text{h}^{-1}$), and the third harmonic – in the region of $10 \text{ m}\cdot\text{s}^{-1}$ ($36 \text{ km}\cdot\text{h}^{-1}$);
4. The maximum amplitude of the wheel set bouncing due to the longitudinal non-uniformity of the track is an order of magnitude higher than the same indicator due to geometric unevenness on the rail tread surface.

The performed analysis allows us to make an assumption that the longitudinal uneven elasticity railway track should be normalized since the longitudinal uneven elasticity of the railway track creates sufficiently large portable accelerations acting on the units of the rolling stock.

Author contributions

All the authors have contributed equally to creation of this article.

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